

# Step on It: Approaches to Improving Existing Vehicles’ Fuel Economy

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## Abstract

Personal transportation seems poised for a transformation as vehicles begin communicating with their surroundings, warning drivers of road hazards, and even driving themselves. These technologies promise to reduce congestion and accidents, potentially by an order of magnitude. In this paper, we argue that these new technologies, along with roadway infrastructure investment, also have the potential to drastically reduce the amount of fuel needed for drivers to complete trips, even without otherwise changing their vehicles. These benefits occur by reducing the substantial variation in fuel economy achieved by different drivers in identical vehicles. We document this variation and show that it somewhat surprisingly does not come from differences in driving styles but from differences in the characteristics of different drivers trips, such as number of stops and amount of time spent idling. Thus technology will not improve fuel economy by forcing drivers to drive more efficiently but rather through its ability to improve traffic flow and reduce stops. This means that the environmental benefits of these technologies are distinct from and in addition to any benefits from more conventional policies like gasoline taxes and fuel economy standards.

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# 1 Introduction

From the self-driving “Google car” to connected vehicle technology, personal transportation is poised for a dramatic transformation. Lane-departure and blind-spot warning and auto-breaking technologies are already installed in many new vehicles, but the cars of the near future will also communicate with each other and the surrounding infrastructure to allow drivers to avoid congestion and accidents. Traffic signals will be able to adjust their timing in response to changing traffic flows. Eventually, we will likely see vehicles that “drive themselves” for at least some parts of trips.

Simultaneously, worldwide fuel consumption and related externalities are a perennial policy issue. Gasoline combustion from vehicles contributes substantially to greenhouse gas emissions and is a major contributor to local air pollution, especially in urban areas. The standard economic policy approach to these externalities would be a gas tax, but the political toxicity of such a tax in the United States has led to gas taxes that are substantially lower than externalities (Parry and Small AER). Instead, US environmental policy for personal vehicles has focused on fuel economy standards such as the Corporate Average Fuel Economy (CAFE) standards, which economists have generally disliked for their inefficiency. Policymakers have even attempted information provision campaigns in order to make fuel economy more psychologically relevant in vehicle purchasers’ decisions. Yet all of the policies are focused on the vehicle fuel economy or number of miles driven rather than *how* the vehicle is driven. For this dimension of fuel use, drivers are told to accelerate more gradually, brake less, or drive slower on the freeway.<sup>1</sup>

In this paper we argue that new technologies and infrastructure investments should be thought of as an under-utilized avenue for environmental policy. These technologies have the potential to reduce fuel consumption of vehicles already on the road and improve drivers’ welfare. This win-win scenario stems from the extensive heterogeneity in on-road fuel economy that exists among drivers of identical vehicles. We use a unique dataset of high-frequency data on drivers’ behavior and fuel consumption to document this heterogeneity: in our sample of 108 drivers driving nearly identical 2006 and 2007 model year Honda Accords for 6 weeks each, the average driver fuel economy ranged from 8.1 to 13.6 liters per 100 kilometers (17.3-29 miles per gallon). This means that these drivers ranged from getting the estimated fuel economy of a 2007 4 wheel drive Honda Pilot to the fuel economy of a 2007 Honda Fit.<sup>2</sup>

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<sup>1</sup>[www.fueleconomy.gov](http://www.fueleconomy.gov)

<sup>2</sup>The 2007 Honda Pilot large SUV in 4 wheel drive with a 3.5 liter, 6 cylinder, 5-speed automatic engine got an EPA-estimated combined fuel economy of 17 mpg while the 2007 Honda Fit 1.5L, 4 cyl, automatic

This difference is greater than the change in the Corporate Average Fuel Economy standard between 1990 and 2025.

Further, we show that this substantial variation in fuel economy is largely coming from *where* people drive rather than *how* they drive: the number of times the driver accelerates from a stop per kilometer explains over three times as much of the variation across drivers in our sample as the speed the driver chooses on the freeway or the rate at which the driver chooses to accelerate. Overall, the number of stops and time spent idling affect fuel economy substantially more than a driver’s “aggressiveness” and yet are often out of a driver’s control on a given trip.

Further supporting the idea that road characteristics are more important than driver aggressiveness in determining fuel use, we pair a behavioral optimization model with estimates from a physical model of fuel use given drivers’ decisions and show that most drivers have values of time that make aggressive driving optimal for reasonable gas prices. Therefore, small increases in gasoline prices, such as those coming from a gas tax, are unlikely to substantially change drivers’ decisions about how aggressively to drive and therefore how much fuel to use on a given stretch of road. This result fits nicely with the disagreement that has existed in the literature as to whether drivers change their driving speed with changing gasoline prices. While Burger and Kaffine (2009) find that higher gas prices did not change drivers’ behavior in Los Angeles (other than through a reduction in congestion), Wolff (2014) finds that drivers in rural Washington state do reduce their speed slightly as gas prices rise. Since these two groups of drivers likely have substantially different values of time, we would expect that Los Angeles drivers are constrained by speed limits, safety considerations, and congestion such that a marginal change in the gas price does not change their behavior, while drivers in rural Washington state are both less constrained and more price sensitive.

If gas taxes are unlikely to substantially affect fuel consumption conditional on the vehicle driven and the trip taken and much of the variation in fuel economy is coming from constraints that are out of a driver’s control, then it is important to understand how reducing those constraints, for instance with connected vehicle technology or investment in roundabouts at major intersections, would change fuel use. To this end we conduct policy simulations based on our empirical model to understand how fuel consumption and travel time would change with a reduction in the number of stops drivers face. We allow drivers on a given trip to optimize not only their driving behavior but also their route selection between an local route with multiple stops and a highway route that is longer but has a higher speed

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5S got an EPA-estimated combined fuel economy of 29 mpg. [www.fueleconomy.gov](http://www.fueleconomy.gov)

limit and no stops. These simulations show that there are two benefits of eliminating stops: drivers on the local route are able to achieve better fuel economy while some drivers switch from the highway route to the local route. This substitution across routes actually *increases* fuel economy as measured in miles per gallon but decreases drivers' total fuel consumption on the trip due to the shorter trip length. This shows that the reallocation of vehicles to more direct routes could be a secondary environmental benefit of policies that smooth traffic flows.

The remainder of the paper is organized as follows: Section 2 describes the data on individual driver behavior and fuel use, documents the variation in fuel economy across drivers in identical vehicles, and provides preliminary evidence that this variation is coming from roadway characteristics rather than driver behavior. Section 3 pairs a behavioral optimization model with an empirical model of fuel consumption and presents results from the estimation of the physical model. Section 4 uses these empirical results to show that for reasonable gas prices, drivers have little incentive to unilaterally improve fuel economy. Section 5 presents policy simulations to show how technology that reduces stops can reduce fuel use while simultaneously reducing travel times. This section also shows how the re-allocation of drivers across routes may increase fuel savings even while decreasing average fuel economy. Section 6 concludes.

## 2 Evidence on the Variation in Fuel Economy

We use a novel engineering dataset containing observations of real-world driving behavior. Between April 2009 and May 2010, the University of Michigan Transportation Research Institute (UMTRI) conducted its Integrated Vehicle-Based Safety Systems (IVBSS) study to test a prototype crash warning system. UMTRI provided 108 drivers with one of 16 identical vehicles to use as their primary vehicle for forty days.<sup>3</sup> Data on nearly 600 variables were collected from vehicle instruments, including fuel consumption, location, speed, radar data on nearby vehicles, and video of both the driver and the road surrounding the vehicle. This high-frequency data provides detailed information about road characteristics and driver behaviors that affect fuel economy.

UMTRI recruited the experimental sample from registered Michigan drivers living in southeast Michigan with no major driving infractions. From respondents to an initial letter,

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<sup>3</sup>There were 117 drivers who entered the study. Nine drivers were removed from the study early because either they were not driving the vehicles enough or they were allowing other people to drive the vehicles. We only use data from the 108 drivers who completed the experiment.

UMTRI selected people who drove more than 12,000 miles per year and who were evenly distributed across gender-age bins. Participants were nominally compensated for their completion of pre- and post-experiment surveys. Drivers received the cars with a full tank of gasoline, but after that they were responsible for additional gasoline purchases. Because study participants drove more than the national average and had relatively clean driving records, we might expect them to be more efficient and less aggressive than the average U.S. driver, which would bias us towards finding less variation in their fuel economy.

The original experiment allowed participants to drive the vehicles for 12 days and then turned on the crash warning system. The system incorporated four types of warning: forward collision, lateral drift, lane departure, and curve speed. An underlying concern was that these warnings might startle the drivers or otherwise exacerbate dangerous driving situations. However, the experiment found high acceptance by drivers of the system and little overall change in driver behavior, with the biggest effect being a reduction in unsignalled lane changes (Sayer et al., 2011). We do not make use of the original experimental design in this study, but instead pool the control and treatment periods for each driver.

We observe 6,352 hours of driving over a total distance exceeding 353,000 kilometers, or 220,000 miles (Table 1). The average distance traveled is 80 kilometers per day, which is about 38 percent greater than the national average.<sup>4</sup> While the speed exceeded 100 km/h for only 20 percent of the driving time, these observations comprised over 40 percent of the distance traveled and 35 percent of the fuel consumed. Time spent idling or moving at less than 5 kilometers per hour also made up 20 percent of time spent in the vehicles, and accounted for 4.3 percent of total fuel consumption. In total, the drivers used nearly 34,000 liters of gasoline (over 9700 gallons).

Overall fuel economy during the experiment was 9.6 L/100 km, or 24.5 miles per gallon (second block of Table 1). The vehicles in the experiment were 2006 and 2007 Honda Accord EX 4-door sedans with a V6 engine. The EPA-reported fuel economy for these vehicles is 13.07 L/100km (18 miles per gallon) for city driving and 9.05 L/100km (26 miles per gallon) for highway driving.<sup>5</sup> Fuel economy was considerably worse at speeds below 50 km/h.

There is substantial variation in fuel economy across the 23,651 trips (Figure 1). Less than half of all trips (46 percent) achieve an average fuel economy between the EPA's highway

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<sup>4</sup>In 2009, the mean daily travel per person in the U.S. was 58.13 kilometers (U.S. Department of Transportation Federal Highway Administration, 2012).

<sup>5</sup>Information obtained from the EPA's fuel economy website at <http://www.fueleconomy.gov>. The methodology for estimating fuel economy was changed for model years 2008 and later. These estimates are based on the new methodology. There is no difference in reported fuel economy between the 2006 and 2007 model years.

and city fuel economy estimates, with 39 percent having a fuel economy worse than the city estimate. Because the EPA’s estimates of highway and city fuel economy are based upon representative drive cycles, we would not expect them to capture the full amount of variation in fuel economy across trips.

Aggregating to the driver level, the EPA’s fuel economy estimates are reasonable bounds on the overall fuel economy that drivers achieve (Figure 2).<sup>6</sup> 16 percent of drivers have an overall fuel economy better than the highway estimate, while 2 percent of drivers have a fuel economy worse than the city estimate. However, the variation in fuel economy across drivers is still striking, with a range from 8.1 to 13.6 L/100 km. This range is equivalent to the difference in EPA average fuel economy between the Toyota Venza (23 mpg) and the Toyota Prius (50 mpg).<sup>7</sup> It is also greater than the change in the Corporate Average Fuel Economy (CAFE) standard between 1990 and 2025 for any footprint size, and nearly 40% greater than the change for large light trucks.<sup>8</sup>

Figure 3 shows the relationship between the driver average fuel economy from Figure 2 and six summary measures that describe driving behavior. These measures are the fraction of time spent idling, the number of acceleration events per kilometer, average speed, average speed conditional on speed being above 100 km/h, average acceleration during acceleration events, and average deceleration during deceleration events.<sup>9</sup> These are converted into z-scores in order to standardize the interpretation, so that a one unit change in the variable corresponds to one standard deviation.

Three of the summary statistics about driving behavior—idle time, acceleration events per kilometer, and average speed—are highly correlated with average fuel economy. More idling, more frequent accelerations, and a lower average speed are associated with higher fuel use.<sup>10</sup> These variables are all correlated with the type of route. Compared to highway

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<sup>6</sup>LeBlanc et al. (2010) provide summary results about variation in fuel economy from the same dataset used in this paper. Figure 1 corresponds to Figure 1 from their paper. They also show the distribution of fuel economy for constant speed highway driving and for acceleration events.

<sup>7</sup>The difference in gasoline consumption between the most and least efficient driver was 5.5 liters per 100 kilometers. The EPA fuel economy for the 2014 Toyota Venza and 2014 Toyota Prius are 10.2 and 4.7 liters per 100 kilometers respectively (23 and 50 miles per gallon), also a difference of 5.5 liters per 100 kilometers. Fuel economy information from <http://www.fueleconomy.gov/feg/findacar.shtml>.

<sup>8</sup>1990 CAFE standards were 27.5 mpg for cars and 20 mpg for light trucks. By 2025 the standards will rise to 60 mpg for the smallest cars and 30 mpg for large light trucks.

<sup>9</sup>Acceleration events are defined as an increase in speed of at least 5 m/s with no more than a 0.001 m/s reduction in speed over any one second interval during the event. Deceleration events are defined similarly for a reduction in speed of at least 5 m/s.

<sup>10</sup>The strong negative correlation between average speed and average fuel economy was noted by Evans (1979), based on field experiments of drivers in urban traffic. He said that average speed could be used as a single statistic to describe the complex characteristics of urban driving.

routes, city routes have frequent stops at traffic lights and stop signs, which leads to idling at the stop, accelerations away from the stop, and a lower average speed.<sup>11</sup> The average rate of acceleration and deceleration, during the acceleration and deceleration event windows, are positively (negatively for deceleration) correlated with fuel consumption. Average speed conditional on speed being above 100 km/h is uncorrelated with average driver fuel consumption.

As further descriptive evidence on the determinants of overall fuel economy, Table 2 shows estimation results for a linear regression of mean fuel consumption on the summary statistics about driving. The first three columns show results from the driver’s entire six-week driving period. Columns 4 and 5 show results using the same variables calculated weekly for each driver. These driver-week regressions include driver fixed effects, so that the coefficients are identified from within-driver changes over time. In all regressions, the explanatory variables (except demographics) are converted to z-scores.

Column 1 includes only factors that are plausibly outside the driver’s control: demographic characteristics, outside temperature, and air conditioning use. Both lower temperatures and greater air conditioning use are associated with higher gasoline consumption. Column 2 adds the three variables related to route characteristics that were shown in Figure 3 to be highly correlated with fuel economy: acceleration events per kilometer, idle time, and average speed. All are statistically significant although the coefficient on average speed switches sign (so higher speed is associated with higher fuel use) compared to the simple correlation in the figure. Adding these three variables increases the  $R^2$  of the regression from 0.21 to 0.87.

Column 3 then adds additional variables related to the driver’s behavior, including the acceleration and deceleration rates, the average speed conditional on being over 100 km/h, and the proportion of driving at speeds above 100 km/h. These have little additional explanatory power. Faster speeds and higher acceleration rates have a statistically significant association with greater fuel use. The coefficient on the number of acceleration events per kilometer has the largest magnitude: a one standard deviation increase in this variable increases fuel use by 0.71 liters per 100 kilometers. By comparison, the factors directly within the driver’s control, such as acceleration rate, have coefficients with much smaller magnitudes.

This result is even stronger for the model with weekly data and driver fixed effects

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<sup>11</sup>Idling, frequent accelerations, and low speeds could also be characteristic of highway driving in stop-and-go traffic. This is less common in our setting than in other metropolitan areas. For the 12 months ending May 2010, Detroit ranked as the 25th most congested metropolitan area in the U.S. and Canada (INRIX, 2010).

(Columns 4 and 5). Based on within-driver variation in driving behavior, a one standard deviation increase in the acceleration rate increases fuel use by 0.14 liters per 100 kilometers. The effect of a one standard deviation increase in the number of acceleration events per kilometer is more than eight times larger. Overall, the results from Table 2 suggest that most of the observed variation in fuel economy does not come from variation in factors that are directly within the driver’s control.

We can use similar regressions to examine the reduced form relationship between fuel economy and gasoline prices. Appendix Table 12 adds the z-score for the mean gasoline price to the regressions in Table 2.<sup>12</sup> Gasoline prices have a small but statistically significant negative effect on fuel consumption in the cross-sectional regressions, even after controlling for driving characteristics such as speed and acceleration. However, for regressions using within-driver variation in gas prices, the effect is small in magnitude, statistically insignificant, and of the wrong sign. This is consistent with results in Burger and Kaffine (2009) who find little change in driver behavior for a change in fuel prices, although we are able to control for driver fixed effects directly. Of course, the amount of variation in gas prices over a 6 week period somewhat limits our willingness to generalize from this result.

Combined, this data suggests that there is substantial variation in fuel economy across drivers of identical vehicles, but that variation is related more to the road conditions on the driver’s trips than the driver’s choice of how aggressively to drive. It appears that drivers may not even be changing their driving behavior in response to gas price changes. In the next section we develop a physical and behavioral optimization model to help understand whether changes in fuel prices via Pigouvian gas taxes are likely to affect fuel economy substantially. We will also use these estimates in our policy simulations of the impact of vehicle technology that can reduce the number of stops a driver faces on a given trip.

### 3 Determinants of Driver Fuel Use

There are two reasons why one driver might get better fuel economy than another: the driver could drive in a way that better minimizes fuel use over a route or the driver could drive routes that demand less fuel. We model the driver,  $i$ , as choosing which route,  $r$ , to take on a given trip,  $\tau$ , between a given origin and destination, and the optimal speed path

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<sup>12</sup>The daily mean gas price is calculated using station-level data from OPIS for stations in the nine counties in south-east Michigan. Stations with missing data on a particular day are excluded from the calculation of the mean price for that day. These daily prices are then aggregated to a mean price for each driver, either weekly or for the six-week driving period.



$\mathbf{s} = \{s_1, \dots, s_T\}$  to complete that route, conditional on obeying traffic laws.

$$\begin{aligned} \max_{r, \mathbf{s}} U_{ir\tau} &= D_{ir\tau} - v_i h(\mathbf{s}|x_{r\tau}) - p_\tau f(\mathbf{s}|x_{r\tau}) - c(\mathbf{s}|x_{r\tau}) \\ &\text{subject to } s_t \leq \mathcal{S}_t \quad \forall t = 1, \dots, T_{r\tau} \end{aligned} \tag{1}$$

Here  $U_{ir\tau}$  is the utility that the driver gets from completing a given trip on a given route with a given speed path.  $D_{ir\tau}$  is the value the driver gets from completing the trip, which is assumed to be large such that drivers always choose to complete the trip.<sup>13</sup> Furthermore,  $v_i$  is driver  $i$ 's value of time,  $h(\mathbf{s}|x_{r\tau})$  is the time it takes to complete a route at a given speed given the characteristics of the route such as pavement quality and the straightness of the road,  $x_{r\tau}$ .  $p_\tau$  is the price of gasoline, which is assumed to be constant across routes, and  $f(\mathbf{s}|x_{r\tau})$  is the fuel consumed on a route given the driver's choice of speed. Finally,  $c(\mathbf{s}|x_{r\tau})$  are additional costs such as safety and vehicle depreciation, that are assumed to vary with the route's characteristics and the driver's speed choices. Importantly, the driver must choose a set of speeds that obey traffic laws, which we represent as a vector of speed limits  $\mathcal{S} = \{\mathcal{S}_1, \dots, \mathcal{S}_T\}$  that include both standard speed limits and constraints on speed such as stop signs, signals and traffic circles.<sup>14</sup>

In order to analyze the effect of policy on driver behavior, we will start by abstracting from the driver's choice over routes and assume that there is only one route on the driver's trip and the driver will only choose speed on that trip (we will add route choice back into the analysis in our simulations in section 5). In this context, the driver chooses the speed vector that sets the marginal benefit of speed (generally reduced travel time) equal to the marginal cost of speed (generally increased fuel consumption and increased safety and depreciation costs). For the sake of this analysis, we will abstract from safety and depreciation costs since our drivers are unlikely to internalize depreciation for vehicles they don't own and safety costs have been considered elsewhere (e.g. Van Benthem (2012)), while still recognizing that these costs exist and are related to the driver's choice of how aggressively to drive.

In order to understand drivers' optimal driving aggressiveness, we must understand how changes in driving behavior influence fuel consumption. In Appendix ??, we present a

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<sup>13</sup>The assumption that  $D_{ir\tau}$  is large precludes substantial changes in the number of trips taken given a policy, but it allows us to focus on changes in behavior conditional on drivers' trips.

<sup>14</sup>Of course, a driver could choose to break speed limit laws and risk getting a ticket. We do not allow for this in our analysis, but as long as speeding and running stop signs or red lights leads to a discontinuous and large increase in costs drivers will choose to obey traffic laws.

theoretical structure of a physical model based on Saerens et al. (2010) and Hellström et al. (2009). This model results in an equation that says that the rate of fuel use is a nonlinear function of velocity, acceleration, and road grade. Empirically, we will approximate this function using the interaction of sixth-order polynomials in velocity, fourth-order polynomials in acceleration, and road grade. We allow additional flexibility by separately modeling positive and negative acceleration. Combining all of these components with our second-by-second data gives us our estimation equation:

$$y_t = \alpha + \sum_{i=0}^6 v_t^i \left[ \sum_{j=0}^4 \beta_{ij0} (a_t^+)^j + \sum_{k=0}^4 \beta_{i0k} (a_t^-)^k + \delta_i \alpha_t \right] + \gamma \mathbf{z}_t + \varepsilon_t \quad (2)$$

In Equation (2),  $y_t$  is the fuel use for a given second of driving in our sample, as measured in liters per 100 kilometers.  $v_t$  is the mean speed during the second-of-sample  $t$ ,  $a_t^+$  is the mean acceleration in meters per second squared if this is positive (and zero otherwise),  $a_t^-$  is the mean acceleration in meters per second squared if this is negative (and zero otherwise), and  $\alpha_t$  is the mean road grade measured in radians.  $z_t$  contains other explanatory variables that are not interacted with the polynomial in speed: the outside temperature in degrees Celsius and a measure for the use of the air conditioner.

Because the dependent variable of equation (2) is fuel use measured in liters per 100 kilometers, this variable approaches infinity for extremely low speeds or idling. For this reason, we estimate a separate model for fuel consumption at zero or very low speeds (less than 5 km/h). This low-speed model is identical to equation (2) except that the dependent variable is measured in milliliters per second.

Table 3 shows the results for equation (2), estimated for all observations with speed above 5 km/h. Column 1 shows the results for a quadratic in speed and excluding all acceleration terms, while Column 2 adds linear terms in acceleration, with positive and negative acceleration entering separately. Column 3 adds the interaction of the linear terms in acceleration and speed and Column 4 shows a selection of coefficients from the full set of estimation results, including the full interactions between a sixth-order polynomial in speed and a fourth-order polynomials in the two acceleration terms.

The full model in column 4 fits the data extremely well and substantially better than other models, with an  $R^2$  of 0.931. The largest increase in  $R^2$  comes from adding the linear acceleration terms to the model, suggesting that acceleration is critically important in understanding fuel use, even controlling for speed. This means that speed sensor data will

be substantially weaker at understanding on-road fuel use than the panel data that we use in this study.

The coefficient estimates are fairly consistent across models at least in terms of sign. Increased speed decreases fuel use, but at a decreasing rate. Positive acceleration increases fuel use substantially across all of the models in table 3, and negative acceleration decreases fuel use in the full model in column 4. The interactions between acceleration and speed show that his effect is largest at low speeds and diminishes at higher speeds. Uphills increase fuel use and downhill decrease fuel use, as shown by the positive coefficient on the sine of road grade, and air conditioner use always increases fuel use. Higher outdoor temperatures generally decrease fuel use, as the engine runs more efficiently at higher temperatures.<sup>15</sup>

The implied minimum constant-speed fuel use occurs 90 kilometers per hour (55.9 miles per hour) for the full model, at which point the vehicle is using 7.14 liters per 100km (32.9 miles per gallon). Figure 5 makes this relationship clear by showing both the observed and predicted fuel economy for constant speed driving at different speeds over 2.5 kilometer per hour speed bins. The gray bars are the observed fuel economy in our data over all one-second observations in our data where acceleration is zero and the vehicle is driving on level road. The black line is the fitted relationship using the coefficients from column 4 of table 3, with acceleration and grade set to zero and all other non-speed terms set to their mean values in the sample. The model fits the data very well and shows that there is a substantial improvement in fuel use as speed increases from very low speeds (and therefore inefficient low gears) and a more gradual increase in fuel use at speeds over 100 kilometers per hour (62 mph). The minimum predicted fuel use from our model is somewhat higher than the observed minimum fuel use at 72.5-75 kilometers per hour (45-46.6 mph), but both the predicted curve and the observed fuel use are very flat through this entire region.

We have seen that the model fits constant speed driving quite well, as evidenced by figure 5. We conduct an additional check of model fit by taking a single trip and looking at the predicted and observed fuel use given the characteristics of the trip.

Figure 6 shows the observed and predicted fuel use over a short trip of just under 6

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<sup>15</sup>Table 4 shows the same set of estimation results, but on observations at speeds less than 2 m/s and a dependent variable of mL/s. In this speed range, increasing speed increases fuel use per second, but at a decreasing rate (although in each second the vehicle is covering a greater distance). Positive acceleration again increases fuel use and negative acceleration decreases fuel use, but this relationship is strongly tied to the speed of the vehicle as the interaction term is large and negative for positive acceleration interacted with speed and large and positive for negative acceleration interacted with speed. As before, air conditioner use increases fuel use and higher outside temperatures decrease fuel use. This model does not fit the data nearly as well as it fits the higher-speed data, with the  $R^2$  only reaching 0.494 for the full model in column 4.

kilometers. The top panel of figure 6 shows the actual fuel use in one-tenth of a kilometer bins with the gray bars. The black diamonds display the predicted fuel use in that one-tenth of a kilometer bin using the characteristics of this particular trip. The bottom panel of figure 6 shows the speed in kilometers per hour over the trip. The obvious first take-away is that the model predicts the variation in fuel consumption over the trip extremely well. The black diamonds are generally very close to the tops of the gray bars, although there are occasionally some small differences. The second thing to notice about figure 6 is that fuel consumption is much higher during acceleration events. The fuel consumption when the vehicle is accelerating is substantially higher than the fuel consumption when the speed is either constant or decreasing. This figure does not make it clear whether different acceleration patterns drastically affect fuel use, but we will explore these questions further in the next section.

As suggested in the introduction, the current primary policy approach to improving drivers' on-road fuel economy is to suggest that drivers should decrease their speed on the freeway and accelerate less aggressively in order to improve fuel economy. The estimates of our physical model suggest that this advice is somewhat true: fuel consumption is minimized at 90 km/h (55.9 mph), which is substantially slower than most people drive on the freeway, and the linear acceleration term does have a positive relationship with fuel use. However, neither of these facts take into account the trade-offs that are central to the behavioral model: increasing fuel consumption by increasing speed or acceleration may be optimal if a driver has a high enough value of time, taking into account any changes in safety. We next solve explicitly for the minimum combination of value of time and value of safety changes that would imply that a driver should drive less aggressively to conserve fuel.

## 4 Drivers' Incentives to Improve Fuel Economy

Instead of determining whether drivers are behaving optimally in our data, we undertake the more generalizable approach of calculating the values of time for which different driving behaviors are optimal. We focus on two common driving situations. First, we consider drivers' incentive to drive at a faster constant speed. Then we will look at acceleration events to understand the values of time for which drivers have an incentive to accelerate more aggressively.

## 4.1 Constant speed driving

As we saw in figure 5, driving faster decreases fuel consumption at less-than-freeway speeds and increases fuel consumption above 90 km/h (55.9 mph). In table 5, we calculate the cost of driving 100 km at different speeds in terms of fuel consumption, fuel cost (at \$3.50 per gallon), and time, and then use these numbers to calculate the cost of time in \$ per hour that makes the driver indifferent between driving that speed or 10 km/h (6.1 mph) slower if the safety and depreciation costs are zero. At speeds below 90 km/h, increasing speed actually decreases fuel use, so all drivers would prefer to drive faster. Above 90 km/h, increasing speed 10 km/h is optimal if the driver's value of time net of non-fuel costs is quite low: 81 cents for the increase to 100 km/h (61 mph), \$3.84 for the increase from 100 km/h to 110 km/h (68.4 mph), and \$7.88 for the increase from 110 km/h to 120 km/h (74.6 mph). Estimating the changes in safety or depreciation from increasing speeds by this amount is beyond the scope of this paper, but these values seem well below the value of time for most American workers.<sup>16</sup> Even for speed increases above 120 km/h, many drivers will find that their values of time net of safety are above the \$11.67/hour cost of increasing to 130 km/h (80.8 mph) or the \$13.61/hour cost of increasing to 140 km/h (87 mph). This suggests that individuals, left to their own devices, have very little incentive to decrease their freeway speed to the fuel-economy-maximizing level.<sup>17</sup>

There is one dimension on which it makes a lot of sense for drivers to adjust their driving behavior to decrease fuel consumption. If a driver at freeway speeds drives a constant speed over a distance, she will use less fuel than if she covers the same distance at the same average speed, but varies her speed over the distance. Table 8 compares the fuel consumption and fuel economy of a driver averaging 30 m/s (67.1 mph) over a 1 km stretch of road driving at a constant speed versus accelerating and then decelerating at a constant rate ( $2 m/s^2$ ) and then finishing the 1 km drive at 30 m/s. While the time spent completing each of these kilometers is the same, the fuel use is strictly increasing in the amount of variation in speed that occurs over the kilometer. This means that there is no value of time that makes it optimal to drive at varying speeds on the highway rather than an average, consistent speed. This suggests that, at least on flat road, cruise control is useful at improving fuel economy,

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<sup>16</sup>An annual income of \$42,500 is equivalent to an hourly wage of \$21.25. Using the standard assumption that the value of time spent driving is equal to one-half of the wage, this puts the average value of time of American workers at \$10.63.

<sup>17</sup>Of course, this ignores the fact that driving faster could put the driver at an increased risk of receiving a speeding ticket. Since this is largely related to the safety cost of driving we consider it to be part of the safety cost of increased speed and abstract from it here.

and congestion that forces the driver to vary speed is costly even if the average speed the driver can travel is unaffected.

## 4.2 Acceleration events

Since our physical model showed that acceleration is at least as important as speed in determining fuel economy, we also look at how drivers' acceleration decisions affect their fuel use and find the minimum value of time for which a driver would choose to accelerate more aggressively. In order to understand general acceleration patterns, we find all acceleration events in our data where drivers either accelerate from 2-15 m/s (4.5-33.6 mph) or from 20-30 m/s (44.7-67.1 mph). We think of the first set of acceleration events as representing acceleration from something close to a stop and the second set as representing merging onto a freeway.

Table 6 displays descriptive statistics for the acceleration and deceleration events in our data, including the fifth and ninety-fifth percentiles. Intuitively, accelerations at higher speeds take up more distance than accelerations at lower speeds. They also take up more time and fuel than the accelerations from very low speeds. Additionally, as expected, deceleration events use very little fuel even though they last nearly as long in terms of both distance and time as acceleration events.

In figure 7 we show how different choices about acceleration from 2-15 m/s (4.5-33.6 mph) affect fuel consumption. There are two conflicting effects to keep in mind. First, accelerating more aggressively requires more torque on the wheels, which can increase fuel consumption. However, by accelerating more aggressively, the driver gets into the higher, more efficient gears more quickly, which decreases fuel consumption. Additionally, since accelerating more aggressively means that the driver hits the 15 m/s speed in a shorter distance, we standardize our comparison by looking at the fuel consumed over the longest acceleration distance, 250 m, which occurs with  $0.5m/s^2$ , assuming that drivers that accelerate more aggressively maintain the constant 15 m/s speed until they reach 250 m. In figure 7, the dark bars represent the amount of fuel used during the acceleration period at different acceleration rates, and the corresponding diamonds represent the predicted fuel use for these accelerations from our model. The light colored bars add the constant-speed driving that allows the driver to reach 250m.

There are two important things to take away from figure 7. First, accelerating aggressively (up to  $2.75 m/s^2$ ) uses substantially less fuel during the acceleration phase than accelerating very slowly, although this fuel savings diminishes with acceleration. This is because, although

the fuel consumption at any second is higher at higher acceleration rates, the time spent accelerating up to 15 m/s is much shorter with more aggressive acceleration. The second point is that this fuel savings is offset by the fact that the top cruising speed of 15 m/s is reached in a shorter distance, so the vehicle needs to drive at a constant speed of 15 m/s for a longer distance. The combined effect is that accelerating more aggressively uses slightly more fuel than accelerating less aggressively, but the difference is quite small.<sup>18</sup>

Of course, accelerating at a slower rate means that it takes substantially longer to cover 250 m than accelerating quickly. Table 7 shows, for different acceleration rates, the fuel consumption, fuel cost and time for accelerating from 2-15 m/s over 250 m (top panel) and for accelerating from 15-25 m/s over 500m (bottom panel). Table 7 also shows the minimum value of time net of non-fuel costs that would be required to make accelerating at that level preferable to accelerating one level more slowly. For accelerations from a near-stop, the minimum value of time net of non-fuel costs never exceeds \$4.76, which means that drivers would need to have an extraordinarily low value of time or be extremely safety- and depreciation- conscious for accelerating less aggressively to be preferable to more aggressive acceleration. For accelerations onto a highway, the optimal approach is to either accelerate somewhat slowly or extremely aggressively. Of course, both of these results assume that the driver is allowed to drive freely after the acceleration event, allowing the decreased time during the acceleration event to translate into a decrease in the total trip time.

Overall most individual drivers appear to have very little incentive to drive more efficiently in a given vehicle on a given route. The exception, of course, is that drivers should refrain from unnecessarily varying their speed over a flat road, which increases fuel use without simultaneously decreasing trip time. The values-of-time required to substantially change drivers' behavior and therefore decrease their fuel consumption suggest that any politically feasible gasoline tax is unlikely to have much impact on the heterogeneity in on-road fuel economy across drivers. Thus it becomes important to understand the potential effect of other policy approaches, such as encouraging technologies and infrastructure investments that smooth traffic flows and decrease stops.

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<sup>18</sup>Akcelik and Biggs (1987) derive optimal acceleration rates, for different final speeds, to minimize fuel consumption during the acceleration period. They find that a hard initial acceleration minimizes fuel use. Hooker (1988) notes that this objective is different to minimizing fuel consumption for driving over a fixed distance with an initial acceleration then cruising at a constant speed. His conclusion was similar to our result: "When the object is to accelerate from rest to cruising speed and then to cruise for some distance while achieving a fixed overall average speed, fuel economy is not very sensitive to the rate of acceleration."

## 5 Policy Simulations

If most of the variation in fuel economy across drivers in identical vehicles is coming from the number of stops a driver faces and the amount of time she spend idling, then traffic-smoothing technologies can be important environmental policy tools. In order to understand the potential for these types of technologies to simultaneously reduce fuel consumption *and* travel times, we set up a stylized simulation of the potential effect of reducing stops on one route a driver could choose to take between a given origin and destination. This setup allows for a better understanding of both the potential time and fuel savings for drivers who take the local route and for drivers who may choose to switch from longer, higher speed routes to shorter, slower routes.

### 5.1 Simulation model

The simulation model uses the behavioral model laid out in equation 1. The driver is modeled as choosing which route to take between an origin and destination and then choosing a speed (and therefore acceleration) path for that route. For the sake of simplicity, we ignore the safety and depreciation costs of driving and focus on just the time and fuel costs of the driver’s decisions. As in equation 1, the only relevant heterogeneity across drivers in our model is in their assumed value of time,  $v_i$ . Drivers with low value of time will be more likely to choose a slower route and to drive in a way that reduces gasoline consumption. For our simulation, we use the distribution of personal hourly earnings for Michigan adults from the Current Population Survey (CPS) in 2012. Following the standard approach in the travel cost literature, we assume that  $v_i$  is half the hourly earnings. Because the CPS data is reported in 40 bins, we only need to calculate the optimal travel behavior once for each of the possible bins. We then weight each of the observations by the percentage of Michigan adults with earnings in that bin in order to calculate the distribution of simulated gasoline consumption for the population.

The two possible routes that we consider for our simulation correspond, in broad terms, to a “city” and a “highway” route between the same origin and destination points. Figure 8 illustrates the differences between the two possible routes. The city route has a shorter distance (10 kilometers) but the maximum allowed speed is lower (60 kilometers per hour in the base case). The city route also has several required stops (five in the base case) where the car is required to come to a complete stop before being able to accelerate again. By comparison, the highway route has a longer distance (15.7 kilometers) but a higher



maximum allowed speed (110 kilometers per hour). There are no intermediate stops for the highway route.

We characterize the driving “style” on each route by three parameters: the maximum speed, the constant rate of acceleration from a stop, and the constant rate of deceleration to a stop. These parameters are sufficient to calculate the entire time path of speed and acceleration along the route. The calculation immediately gives the total time for the trip. Using the time path of speed and acceleration (in steps of one hundredth of a second), combined with the vehicle fuel consumption model from Section [3], we predict the gasoline consumption over the trip. Total trip cost can then be calculated using an assumed gasoline price and value of time.

For each possible value of time, and for the two possible routes, we choose the values for the speed and acceleration parameters that minimize the total travel cost. We do this using a grid search over 21 possible maximum speeds (in 0.5 m/s increments), 14 acceleration rates (in 0.2 m/s<sup>2</sup> increments) and 14 deceleration rates (also in 0.2 m/s<sup>2</sup> increments). After calculating the minimum travel cost on each route, each driver is assumed to choose the route (city or highway) with the lowest travel cost.

The usefulness of the simulation model is that it allows us to compare various policies aimed at reducing gasoline consumption. These include increases in the gasoline tax (modelled by increasing the gasoline price), reductions in the speed limit (modeled by changing the constraint on maximum speed), and reductions in the number of city stops. Although the model is highly stylized, it does incorporate several realistic features: the heterogeneity across drivers in value of time, the trade-off between time and gasoline consumption estimated from real-world vehicle data, and the ability of drivers to choose between alternative routes to reach the same destination.

## 5.2 Simulation results and policy comparisons

The first column of Table 9 shows the base case results for the simulation. Out of the representative population of Michigan adult drivers, 40.2% chose the city route and 59.8% chose the highway route. With a lower speed and more stops, the city route took a longer time than the highway route: 10.80 minutes compared to 8.75 minutes. However, the advantage of the city route is that gasoline consumption is lower: 1.065 liters compared to 1.330 liters on the highway. The drivers choosing the city route are the ones for whom it is optimal to take an extra two minutes in order to save about a quarter of a liter of gasoline. These are the drivers with the lowest value of time. The drivers with the highest value of time will

always choose the faster highway route that uses more gasoline.

Interestingly, although the city route uses less gasoline, the observed fuel economy in liters per 100 kilometers is lower for the city than for the highway. This is because the additional fuel consumption on the highway route is less than the proportional increase in trip distance. This suggests that a focus on comparison of fuel economies across drivers can be misleading. Because the benefit that drivers obtain from either route is identical—transportation between the same origin and destination—the most relevant comparison is the fuel use for the entire trip, not the fuel use per unit of distance.

The bottom of the table shows the calculation of the overall average cost to the drivers and society of the trip. The driver cost includes the gasoline cost (including gasoline tax if included) and the value of the driver’s time. The average cost of the trip is \$2.92, comprising \$1.16 of gasoline cost and \$1.76 for the value of time. The social cost also includes the value of the carbon dioxide externality from the gasoline consumption, assuming a social cost of carbon of \$40 per ton of carbon dioxide.<sup>19</sup> In the base case, the carbon externality is about 11.4 cents per trip.

The remaining four columns of Table 9 show the effect of imposing different levels of a gasoline tax. The first tax considered is exactly the Pigouvian tax corresponding to the social cost of carbon in gasoline. The remaining three cases consider higher taxes: 25, 50 and 75 cents per liter.

All of the gas taxes cause drivers to shift from the highway to the city route. The drivers that change their route are exactly those drivers whose value of time leaves them only slightly better off on the highway (with shorter time but more gasoline consumption). A slight increase in the gasoline price causes these marginal drivers to switch to the route with lower gasoline consumption.

The higher gas tax leads to only a small change in driving behavior (the choice of speed and acceleration on a given route). On the city route, gasoline consumption declines from 1.065 liters for the trip to 1.063 liters for the trip, when the gas tax increases to 75 cents per liter. This understates the change in behavior of individual drivers because the composition of the city drivers is also changing: the drivers with higher value of time who switch from the highway route are less likely to reduce their speed or acceleration in order to save fuel.

Overall the gas taxes reduce the average fuel consumption for the trip: from 1.223 liters to 1.144 liters. However, because more drivers are choosing the slower route (and, to a

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<sup>19</sup>In the current analysis we do not consider other externalities such as local pollution, traffic congestion, or accidents. Additionally, we do not currently consider the fact that a single gallon of gasoline could have very different overall emissions of local air pollutants as discussed in Fullerton and West (2010).

lesser extent, are choosing to drive slower), the average time for the trip increases from 9.58 minutes to 10.25 minutes. This increase in trip time more than offsets the reduction in gasoline consumption: social cost for the trip is \$3.055 with a 75 cents per liter gas tax, compared to \$3.035 in the base case. Imposing the Pigouvian gasoline tax reduces the social cost by 0.1 cents.

Table 10 shows the effect of changing the city or highway speed limits.<sup>20</sup> The first column in the table repeats the base case results from Table 9. Reducing the city or the highway speed limit by 10 kilometers per hour increases the average trip cost for the drivers by 2 cents (for the lower city limit) and 11 cents (for the lower highway limit). The lower speed limit in the city causes marginal drivers to switch to the highway route, which is faster but consumes more gasoline. Conversely, the lower speed limit on the highway causes marginal drivers to switch to the city route, which uses less gasoline but is much slower. The result that lower speed limits makes drivers worse off in our model is not surprising. Drivers are already assumed to choose their optimal speed, acceleration and route, and could have already chosen the slower speed if it had a lower cost. Any additional constraints on the driver's behavior will necessarily make them worse off, but the difference between a speed limit change on the city route rather than the highway route is that the welfare losses are in terms of increased fuel use rather than increased time.

The table also shows the effect of increasing the city or the highway speed limits. Either policy change would make the drivers in our model better off. A higher city speed limit would lead to a large increase in the number of drivers on the city route. Average fuel consumption for the city drivers would increase by 0.039 liters but this is more than offset by a reduction in the average city travel time by 1.24 minutes. A higher highway speed limit would not change the number of drivers on the highway route, because the marginal driver on the city route has a value of time sufficiently low that they would not drive at the (new or old) highway speed limit anyway. Instead, the higher speed limit on the highways benefits the existing highway drivers who save more than half a minute at the expense of an additional 0.08 liters of fuel consumption. Again, this reinforces the idea that policies that make the more efficient city route more attractive to drivers (by making it take less time or use less fuel) leads to an overall decrease in fuel use relative to policies that make the highway route more attractive.

Interestingly, there is little change in the value of the carbon externality for any of the

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<sup>20</sup>Of course, since these analyses do not account for changes in accident rates, these changes in speed limits should not be seen as a full welfare analysis of changing speed limits. For a more thorough discussion of the safety impacts of speed limit changes, see van Benthem (2012).

possible changes in speed limit. It increases by, at most, 1 cent per trip. This means that the overall change in social cost is dominated by the changes in the drivers' private costs: the gasoline cost and the value of time. In the context of this model, reducing speed limits makes society unambiguously worse off, whereas raising speed limits makes society better off.

Our final policy simulation is to analyze the effect of either reducing the number of stops on the city route or turning all of the stops into traffic circles or "rotaries" where drivers must decrease their speed to 25 km/h in order to pass through the circle, but are not required to stop. Column 1 of Table 11 again presents the same base case as before, with 5 full stops on the city route. Columns 2 through 4 of Table 11 reduce the number of stops to 4, 3, or 2 respectively. As the number of stops on the city route decreases, both the fuel consumption and the trip time on the city route decrease, making the city route more attractive to marginal drivers. This means that the overall private and social trip cost declines, even though the average time cost increases as drivers switch from the quicker highway route to the slower city route. The final column of Table 11 shows the effect of converting all of the city stops to traffic circles, which is roughly similar in aggregate to reducing the number of stops from 5 to 3.

### 5.3 Aggregate Benefits

Using the simulation results on replacing stops with roundabouts, we can calculate the welfare impact of installing a roundabout.<sup>21</sup> lays out estimates of the cost of installing roundabouts and their potential vehicle capacity. In 2014 dollars, an average roundabout costs approximately \$363,000 to install and last for 25 years.<sup>22</sup> Operating and maintenance costs of roundabouts are greater than unsignalized intersections but less than unsignalized intersections, so we will conservatively assume that they are the same as the traffic control devices they replace (which are likely signals). Conservatively, a one lane roundabout has a daily capacity of 20,000 vehicles and a two lane roundabout has a daily capacity of 40,000 vehicles, which we will further conservatively means that each roundabout serves 12,000 vehicles per day.

Our simulations show that replacing a stop with a roundabout decreases the fuel costs of drivers who currently take that route by 2.27 cents. This means that the present discounted value of replacing the stop with a roundabout is \$993,000, which does not include accident

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<sup>21</sup>Federal Highway Administration FHWA-RD-00-067 Roundabout: An Informational Guide, June 2000

<sup>22</sup>This figure does not include land acquisition costs.

costs, which are often cited as the primary reason for installing a roundabout, time savings or unpriced fuel and noise externalities. The number of roundabouts in the US has doubled in the last decade,<sup>23</sup> and our conservative calculations suggest that these 2,500 new roundabouts will save 1.78 billion gallons of gasoline over their 25 year lifecycles.

One downside of roundabouts is that they do take up more land than a standard signalized intersection. This is where the potential for connected vehicle technology becomes evident. If self-driving cars or connected vehicle technology could allow vehicles to travel through many intersections safely without fully stopping then the potential fuel and time savings would dwarf these savings from installing roundabouts.

## 6 Conclusion

Taken together, our results show that although there is a substantial variation in fuel economy across drivers in identical vehicles, this variation is coming from differences in *where* people drive rather than *how* they drive. This result makes sense in the context of our physical and behavioral model which suggest that there is little incentive for most drivers to reduce the aggressiveness of their driving in order to save fuel for reasonable fuel prices. However, these results do strongly suggest that policies that encourage vehicle-to-vehicle and vehicle-to-infrastructure communication in order to smooth traffic flows and reduce the number of stops required on a given route could both decrease fuel consumption and improve driver welfare.

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<sup>23</sup>Eric A. Taub “As Americans Figure Out the Roundabout, It Spreads Across the U.S.” New York Times 7/30/2015

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**Table 1:** Summary statistics for all driving, by speed category

	Speed bin (km/h)						Total
	0	0–5	5–20	20–50	50–100	$\geq 100$	
<b>Total by speed bin</b>							
Time (hours)	1,025	240	492	1,107	2,231	1,257	6,352
% of total	16.1	3.8	7.7	17.4	35.1	19.8	100.0
Distance (km)	0	502	6,134	39,594	161,146	146,028	353,404
% of total	0.0	0.1	1.7	11.2	45.6	41.3	100.0
Fuel consumption (L)	905	539	1,777	5,721	13,112	11,860	33,913
% of total	2.7	1.6	5.2	16.9	38.7	35.0	100.0
<b>Mean by speed bin</b>							
Fuel economy (L/100 km)	.	107.3	29.0	14.4	8.1	8.1	9.6
Speed (km/h)	0.0	2.1	12.5	35.8	72.2	116.2	55.6
Acceleration $> 0$ (m/s <sup>2</sup> )	0.00	0.24	0.42	0.37	0.14	0.07	0.17
A/C usage	0.27	0.26	0.25	0.22	0.20	0.22	0.23
Outside temperature (°C)	13.9	14.8	14.7	14.2	13.6	13.7	13.9



**Table 2:** Estimates for mean driver fuel consumption in liters per 100 kilometers

	(1)	(2)	(3)	(4)	(5)
Acceleration events per km		0.91 (0.144)	0.71 (0.151)	1.29 (0.065)	1.15 (0.071)
Idle time (0–1)		0.49 (0.069)	0.35 (0.086)	0.60 (0.063)	0.44 (0.077)
Speed (km/h)		0.23 (0.126)	-0.05 (0.221)	0.17 (0.068)	-0.29 (0.144)
Speed > 100 km/h			-0.06 (0.114)		0.30 (0.089)
Speed   Speed > 100 km/h			0.20 (0.065)		0.01 (0.035)
Acceleration (m/s <sup>2</sup> )			0.17 (0.047)		0.14 (0.054)
Deceleration (m/s <sup>2</sup> )			0.03 (0.046)		0.01 (0.054)
Outside temperature (°C)	-0.32 (0.182)	-0.39 (0.067)	-0.43 (0.054)	-0.61 (0.072)	-0.60 (0.069)
Air conditioning (0–1)	0.05 (0.180)	0.25 (0.050)	0.27 (0.044)	0.43 (0.049)	0.40 (0.046)
Female (0/1)	0.73 (0.200)	-0.01 (0.081)	0.15 (0.080)		
Age 40–50 (0/1)	-0.69 (0.274)	-0.23 (0.100)	-0.01 (0.111)		
Age 60–70 (0/1)	-0.64 (0.232)	-0.17 (0.101)	0.05 (0.127)		
Constant	10.19 (0.190)	10.25 (0.070)	10.02 (0.098)	10.32 (0.000)	10.31 (0.004)
Driver fixed effects	N	N	N	Y	Y
Observations	108	108	108	758	713
Adjusted R <sup>2</sup>	0.211	0.870	0.897	0.867	0.847

*Note:* Each observation is a driver (Columns 1–3) or a driver-week (Columns 4–5). The dependent variable is the mean fuel economy for the driver over the corresponding period. All explanatory variables (except the demographic indicators) are standardized z-scores for the mean value of the variable over the corresponding period. Means and standard deviations for the variables are shown in Table ??.

**Table 3:** Estimates for fuel consumption in liters per 100 kilometers, at speeds above 1 meter per second

	(1)	(2)	(3)	(4)
Speed (m/s)	-2.23 (0.002)	-1.62 (0.001)	-1.26 (0.001)	-7.77 (0.123)
Speed squared	0.04 (0.000)	0.03 (0.000)	0.03 (0.000)	0.82 (0.021)
Acceleration > 0 (m/s <sup>2</sup> )		23.41 (0.006)	28.51 (0.014)	51.04 (2.308)
Acceleration < 0 (m/s <sup>2</sup> )		3.87 (0.003)	-0.03 (0.008)	-11.94 (2.136)
(Acceleration > 0) × speed			-0.47 (0.001)	-7.33 (1.408)
(Acceleration < 0) × speed			0.34 (0.001)	8.31 (1.087)
Sin of road grade	34.86 (0.235)	51.72 (0.213)	50.63 (0.208)	6.77 (1.714)
Air conditioner (0–1)	1.83 (0.007)	1.67 (0.003)	1.66 (0.003)	1.62 (0.002)
Outside temperature (°C)	-0.02 (0.000)	-0.03 (0.000)	-0.03 (0.000)	-0.03 (0.000)
Constant	34.30 (0.018)	22.88 (0.008)	19.37 (0.010)	37.41 (0.231)
Higher order interactions	N	N	N	Y
Minimum fuel speed (km/h)	91.5	83.6	80.2	90.0
Minimum fuel usage (L/100 km)	5.97	4.00	5.25	7.16
Observations	18,037,794	18,037,794	18,037,794	18,037,794
Adjusted R <sup>2</sup>	0.263	0.881	0.892	0.932

*Note:* Each observation is the fuel consumption, in liters per 100 kilometers, during one second of driving at a speed greater than one meter per second. The regression in the fourth column includes the full set of interactions between a sixth-order polynomial in speed and fourth-order polynomials in positive and negative acceleration as well as the sine of the road grade. The speed that minimizes fuel usage per kilometer, assuming zero acceleration and zero grade, is shown at the bottom of the table, along with the fuel consumption at this speed. Robust standard errors are shown in parentheses.

**Table 4:** Estimates for fuel consumption in milliliters per second, at speeds below 1 meter per second

	(1)	(2)	(3)	(4)
Speed (m/s)	0.27 (0.001)	-0.05 (0.001)	0.01 (0.001)	0.60 (0.042)
Speed squared	-0.07 (0.001)	0.05 (0.001)	-0.00 (0.001)	-1.45 (0.307)
Acceleration > 0 (m/s <sup>2</sup> )		0.54 (0.001)	0.26 (0.001)	0.08 (0.044)
Acceleration < 0 (m/s <sup>2</sup> )		-0.06 (0.000)	-0.13 (0.001)	-0.32 (0.031)
(Acceleration > 0) × speed			0.24 (0.001)	-3.67 (0.134)
(Acceleration < 0) × speed			0.06 (0.001)	3.92 (0.187)
Air conditioner (0–1)	0.12 (0.000)	0.12 (0.000)	0.12 (0.000)	0.12 (0.000)
Outside temperature (°C)	-0.00 (0.000)	-0.00 (0.000)	-0.00 (0.000)	-0.00 (0.000)
Constant	0.51 (0.000)	0.51 (0.000)	0.51 (0.000)	0.51 (0.000)
Higher order interactions	N	N	N	Y
Observations	4,828,216	4,828,216	4,828,216	4,828,216
Adjusted R <sup>2</sup>	0.174	0.456	0.478	0.492

*Note:* Each observation is the fuel consumption, in milliliters, during one second of idling or driving at a speed less than one meter per second. The regression in the fourth column includes the full set of interactions between a sixth-order polynomial in speed and fourth-order polynomials in positive and negative acceleration. Robust standard errors are shown in parentheses.

**Table 5:** Fuel cost and value of time for constant-speed highway driving

	Constant speed (km/h)							
	70	80	90	100	110	120	130	140
<b>Cost per 100 km</b>								
Fuel consumption (L)	7.44	7.27	7.16	7.25	7.63	8.27	9.08	9.90
Fuel cost (\$)	6.87	6.72	6.62	6.71	7.05	7.65	8.40	9.16
Time (minutes)	85.7	75.0	66.7	60.0	54.5	50.0	46.2	42.9
<b>Effect of increasing speed</b>								
$\Delta$ Fuel cost (\$)	.	-0.16	-0.10	0.09	0.34	0.60	0.75	0.76
$\Delta$ Time (minutes)	.	-10.7	-8.3	-6.7	-5.5	-4.5	-3.8	-3.3
Cost of time (\$/hour)	.	-0.88	-0.70	0.77	3.78	7.86	11.76	13.78

*Note:* The first block shows the fuel consumption, fuel cost, and time taken to drive 100 kilometers at constant speeds. Fuel cost assumes a gasoline price of \$3.50 per gallon. The second block shows the effect of increasing speed from the previous column by 10 km/h. For example, increasing speed from 100 to 110 km/h will increase the fuel cost by \$0.38 but reduce the trip time by 5.5 minutes.

**Table 6:** Summary statistics for acceleration and deceleration events

	Acceleration events (m/s)			Deceleration events (m/s)		
	All	2–15	20–30	All	15–2	30–20
<b>Distance (m)</b>						
P5	20.9	47.0	125.1	20.2	39.0	94.2
Mean	140.8	82.8	309.9	93.2	70.4	214.6
P95	381.2	137.6	568.7	227.0	114.3	375.9
<b>Time (s)</b>						
P5	5.1	5.4	5.0	4.2	4.5	3.7
Mean	12.0	8.9	12.2	9.1	7.9	8.4
P95	22.7	14.0	22.2	16.8	12.5	14.9
<b>Fuel used (mL)</b>						
P5	10.0	27.0	58.0	0.0	0.8	0.0
Mean	36.3	32.0	73.3	3.0	2.4	0.2
P95	80.6	38.8	95.2	7.0	5.4	1.0
<b>Acceleration (m/s<sup>2</sup>)</b>						
P5	0.48	0.93	0.45	-1.96	-2.88	-2.68
Mean	1.03	1.60	1.00	-1.25	-1.80	-1.41
P95	1.67	2.44	2.02	-0.69	-1.04	-0.67
Observations	234119	50074	2020	229251	40137	999

*Note:*

**Table 7:** Fuel cost and value of time for acceleration events

	Acceleration (m/s <sup>2</sup> )					
	0.5	1.0	1.5	2.0	2.5	3.0
<b>2–15 m/s over 250 m</b>						
Fuel consumption (mL)	47.12	45.26	45.16	45.66	46.20	46.74
Fuel cost (cents)	4.36	4.18	4.18	4.22	4.27	4.32
Time (seconds)	27.9	22.3	20.4	19.5	18.9	18.5
$\Delta$ Fuel cost (cents)	.	-0.17	-0.01	0.05	0.05	0.05
$\Delta$ Time (seconds)	.	-5.6	-1.9	-0.9	-0.6	-0.4
Cost of time (\$/hour)	.	-1.10	-0.18	1.77	3.18	4.76
<b>15–25 m/s over 500 m</b>						
Fuel consumption (mL)	71.31	76.71	79.80	80.96	80.54	79.10
Fuel cost (cents)	6.59	7.09	7.38	7.49	7.45	7.31
Time (seconds)	24.0	22.0	21.3	21.0	20.8	20.6
$\Delta$ Fuel cost (cents)	.	0.50	0.29	0.11	-0.04	-0.13
$\Delta$ Time (seconds)	.	-2.0	-0.7	-0.3	-0.2	-0.1
Cost of time (\$/hour)	.	9.00	15.35	11.63	-6.89	-34.27

*Note:* Each block shows the simulated fuel consumption and time to drive a fixed distance, with an initial acceleration period. Each column illustrates the effect of a different value of acceleration for this initial period.

**Table 8:** Additional fuel cost from highway speed fluctuations, for constant average speed

	Variation in speed (m/s)					
	0	2	4	6	8	10
Distance (km)	1.00	1.00	1.00	1.00	1.00	1.00
Time (min)	0.56	0.56	0.56	0.56	0.56	0.56
Fuel consumption (L)	0.075	0.085	0.095	0.105	0.117	0.129
L/100 km	7.46	8.48	9.50	10.54	11.65	12.91

*Note:* The table shows fuel consumption for driving on a 1 km highway segment at an average speed of 30 m/s. There is an initial period of acceleration and deceleration (at a rate of 2 m/s<sup>2</sup>) followed by the constant speed of 30 m/s for the remaining distance. The total difference between the maximum and minimum speed is shown at the top of the column.

**Table 9:** Policy simulation results: Effect of gasoline tax

	Base	Gas tax (cents/liter)			
		9.4	25	50	75
Share on City Route	0.402	0.456	0.517	0.619	0.699
Fuel consumption (L)	1.223	1.209	1.193	1.165	1.144
City	1.065	1.065	1.065	1.064	1.063
Highway	1.330	1.330	1.330	1.330	1.330
Time (minutes)	9.58	9.69	9.83	10.07	10.25
City	10.80	10.80	10.83	10.88	10.89
Highway	8.75	8.75	8.75	8.75	8.75
Fuel economy (L/100km)	9.343	9.460	9.592	9.809	9.980
City	10.651	10.649	10.645	10.637	10.632
Highway	8.753	8.753	8.753	8.753	8.753
Cost components (\$)					
(1) Gasoline cost	1.16	1.15	1.13	1.11	1.09
(2) Gasoline tax	0.00	0.11	0.30	0.58	0.86
(3) Social cost of carbon	0.11	0.11	0.11	0.11	0.11
(4) Value of time	1.76	1.77	1.79	1.83	1.86
Driver cost (1+2+4)	2.921	3.034	3.221	3.516	3.805
Social cost (1+3+4)	3.035	3.034	3.035	3.043	3.055

*Note:* Base case uses average gas price in Michigan in 2012 (\$3.60 per gallon). The other four columns show the effect of a gasoline tax of 9.4, 25, 50 and 75 cents per liter. The first of these (9.4 cents per liter) corresponds to the social cost of the carbon content in one liter of gasoline (based on \$40 per ton of carbon dioxide).



**Table 10:** Policy simulation results: Changes to speed limits

	Base	City (km/h)		Hwy (km/h)		Hwy 100
		50	70	100	120	City 50
Share on City Route	0.402	0.177	0.828	0.565	0.402	0.216
Fuel consumption (L)	1.223	1.270	1.143	1.150	1.272	1.207
City	1.065	1.024	1.104	1.066	1.065	1.024
Highway	1.330	1.323	1.330	1.258	1.411	1.258
Time (minutes)	9.58	9.52	9.42	10.29	9.23	10.35
City	10.80	12.81	9.56	10.76	10.80	12.81
Highway	8.75	8.81	8.75	9.67	8.18	9.68
Fuel economy (L/100km)	9.343	8.745	10.598	9.509	9.654	8.490
City	10.651	10.234	11.042	10.661	10.651	10.235
Highway	8.753	8.812	8.753	9.673	8.175	9.676
Cost components (\$)						
(1) Gasoline cost	1.16	1.21	1.09	1.09	1.21	1.15
(2) Gasoline tax	0.00	0.00	0.00	0.00	0.00	0.00
(3) Social cost of carbon	0.11	0.12	0.11	0.11	0.12	0.11
(4) Value of time	1.76	1.74	1.78	1.94	1.66	1.91
Driver cost (1+2+4)	2.921	2.942	2.868	3.028	2.866	3.060
Social cost (1+3+4)	3.035	3.061	2.975	3.136	2.985	3.173

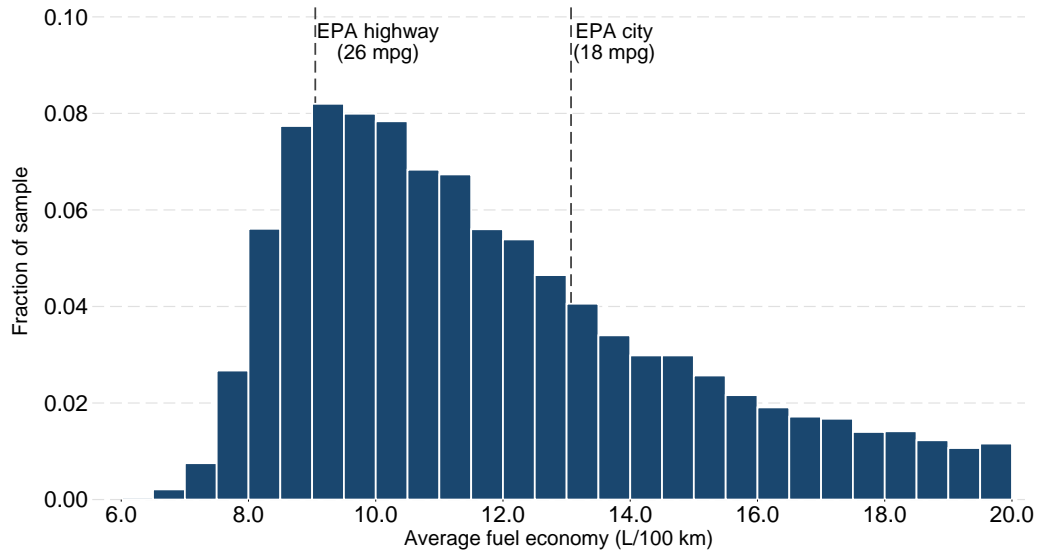
*Note:* Base case (identical to previous table) assumes a city speed limit of 60 km/h and a highway speed limit of 110 km/h. The four city and highway columns show the effect of lowering and raising the speed limit for either city or highway, in 10 km/h increments, from these base levels. The final column shows the effect of lowering the speed limit on both the city and highway at the same time.

**Table 11:** Policy simulation results: Reducing city stops

	Base	Number of stops			Traffic circles
		4	3	2	
Share on City Route	0.402	0.517	0.619	0.699	0.565
Fuel consumption (L)	1.223	1.168	1.107	1.044	1.110
City	1.065	1.018	0.970	0.921	0.942
Highway	1.330	1.330	1.330	1.330	1.330
Time (minutes)	9.58	9.73	9.82	9.89	9.95
City	10.80	10.64	10.47	10.38	10.87
Highway	8.75	8.75	8.75	8.75	8.75
Fuel economy (L/100km)	9.343	9.350	9.228	8.982	9.003
City	10.651	10.178	9.699	9.206	9.418
Highway	8.753	8.753	8.753	8.753	8.753
Cost components (\$)					
(1) Gasoline cost	1.16	1.11	1.05	0.99	1.05
(2) Gasoline tax	0.00	0.00	0.00	0.00	0.00
(3) Social cost of carbon	0.11	0.11	0.10	0.10	0.10
(4) Value of time	1.76	1.79	1.81	1.83	1.81
Driver cost (1+2+4)	2.921	2.895	2.864	2.826	2.865
Social cost (1+3+4)	3.035	3.004	2.968	2.924	2.969

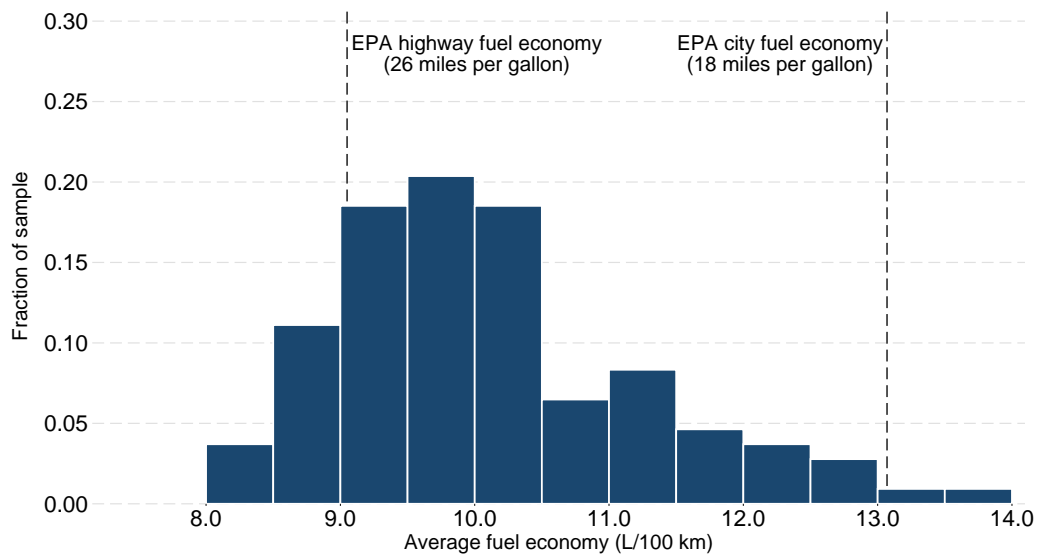
*Note:* Base case (identical to the previous tables) assumes there are five stops on the city route (and none on the highway route). The middle three columns shows the effect of reducing the number of city stops to four, three and two. The final column shows the effect of replacing the five city stops with five traffic circles, in which traffic slows to about 25 km/h to pass through the circle.

**Figure 1:** Distribution of mean fuel economy across trips



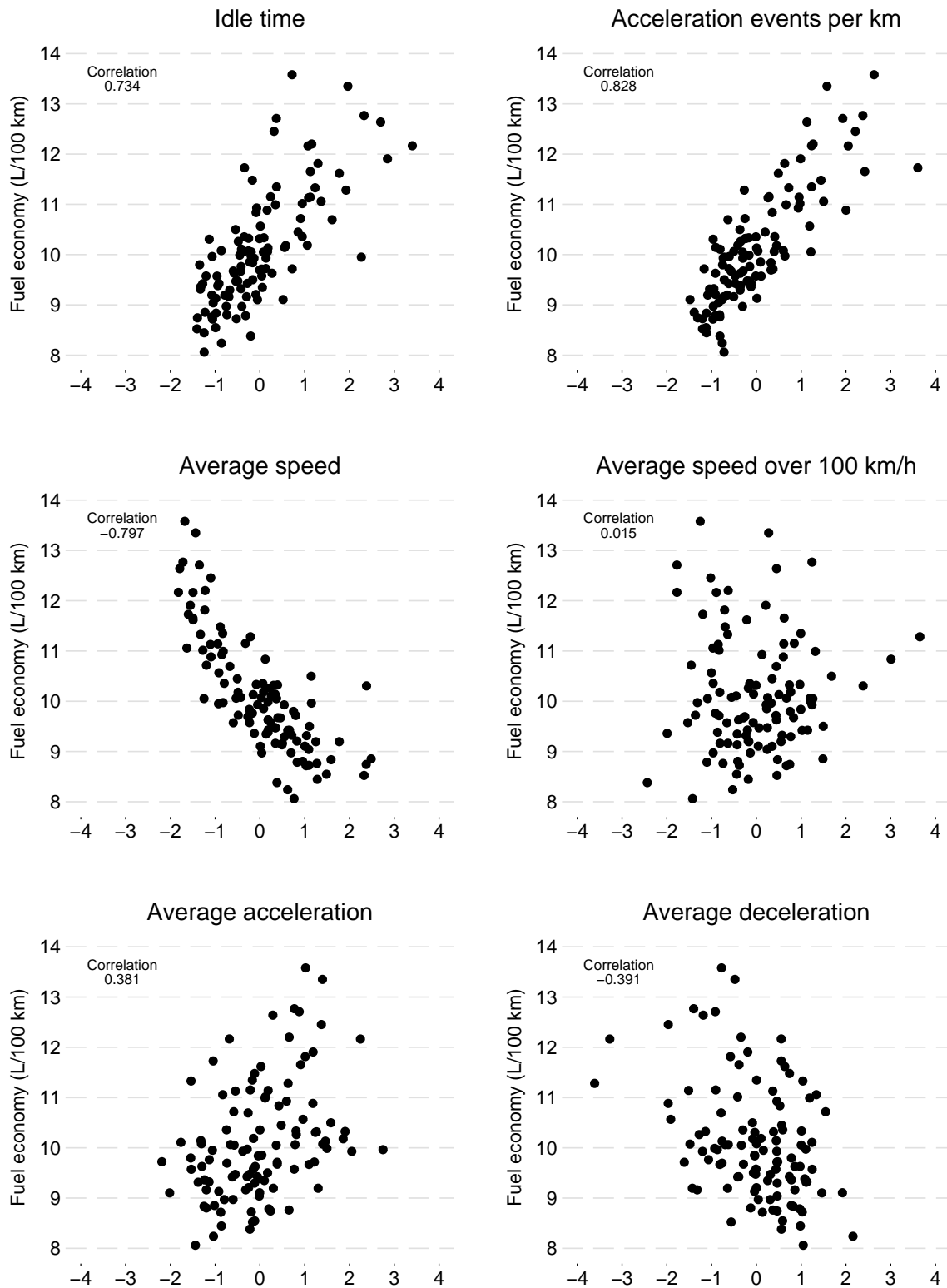
*Note:* The distribution is truncated at 20 L/100 km (11.8 miles per gallon). There are 3,641 trips (13.6 percent of the total) with a fuel economy worse than 20 L/100 km.

**Figure 2:** Distribution of mean fuel economy across drivers



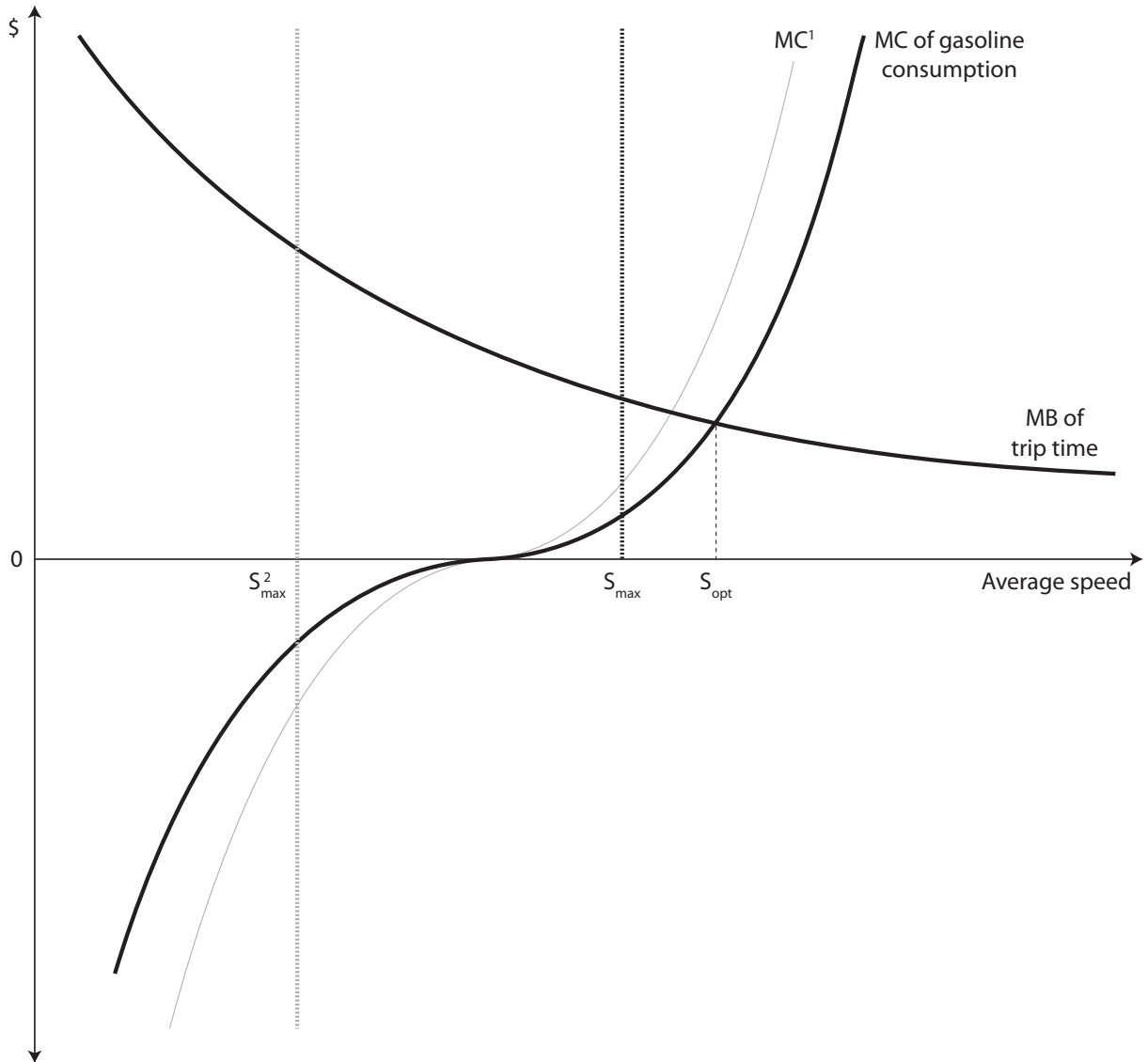
*Note:*

**Figure 3:** Pairwise relationships between driving characteristics and driver average fuel use



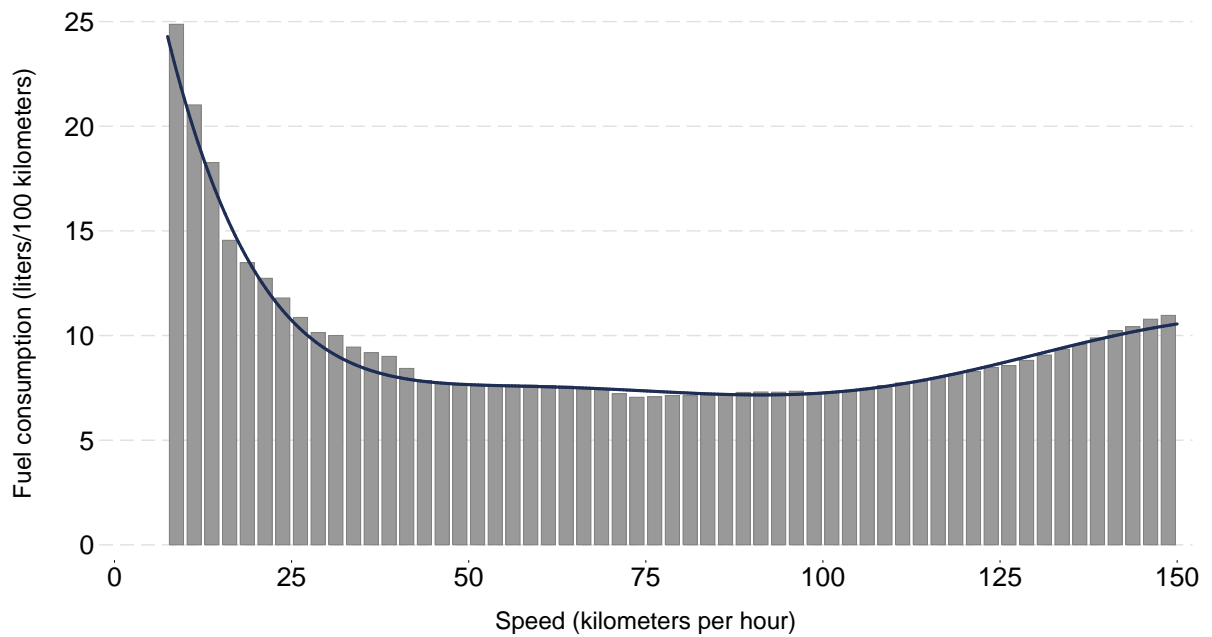
*Note:*

**Figure 4:** Graphical Representation of Drivers' Behavioral Model



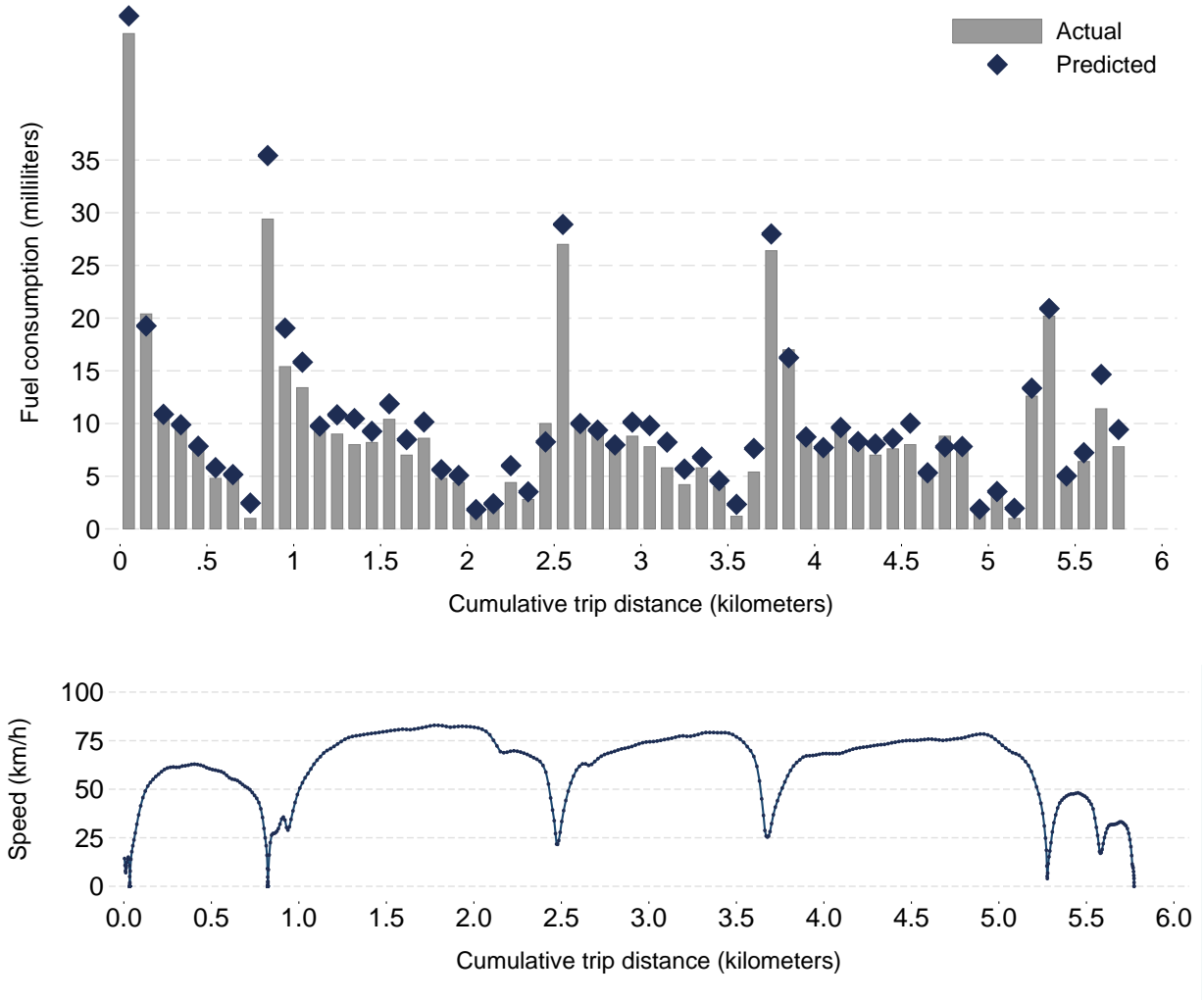
*Note:* Each driver chooses speed to set the marginal benefit of trip time,  $MB$ , equal to the marginal cost of gasoline consumption,  $MC$ , unless constrained by speed limits, stop signs, or traffic signals, represented by  $S_{max}$  and  $S_{max}^2$ .

**Figure 5:** Observed and Predicted Relationship between Fuel Consumption and Speed



Notes: The gray bars show the mean fuel consumption for level driving at constant speed (defined as the absolute value of the grade less than 0.01 radians and the absolute value of acceleration less than  $0.25 \text{ m/s}^2$ ), using 2.5 kilometer/hour speed bins. The function plot shows the predicted relationship between speed and fuel consumption, assuming zero acceleration and zero grade, based on the estimates in Table ??.

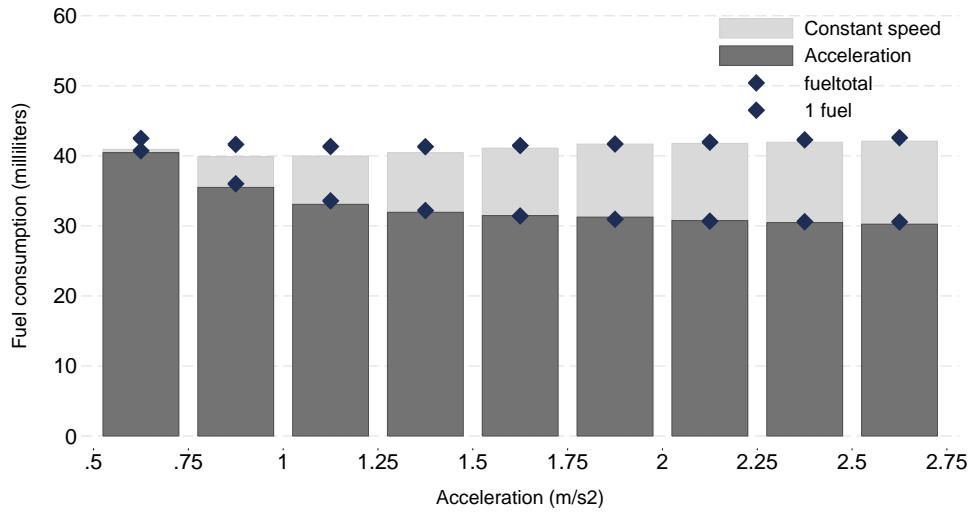
**Figure 6:** Observed and predicted fuel consumption and observed speed for a single trip



*Note:* The top figure shows the fuel consumption during one particular trip. The gray bars show the actual fuel consumption for each 100 meter segment of the trip. The diamond markers show the predicted fuel consumption based on the estimates in Table ???. The vertical axis shows fuel consumption in milliliters for the 100 meter segment; this also corresponds to fuel economy in L/100 km. The bottom figure shows the trace of speed (in kilometers per hour) for the same trip.

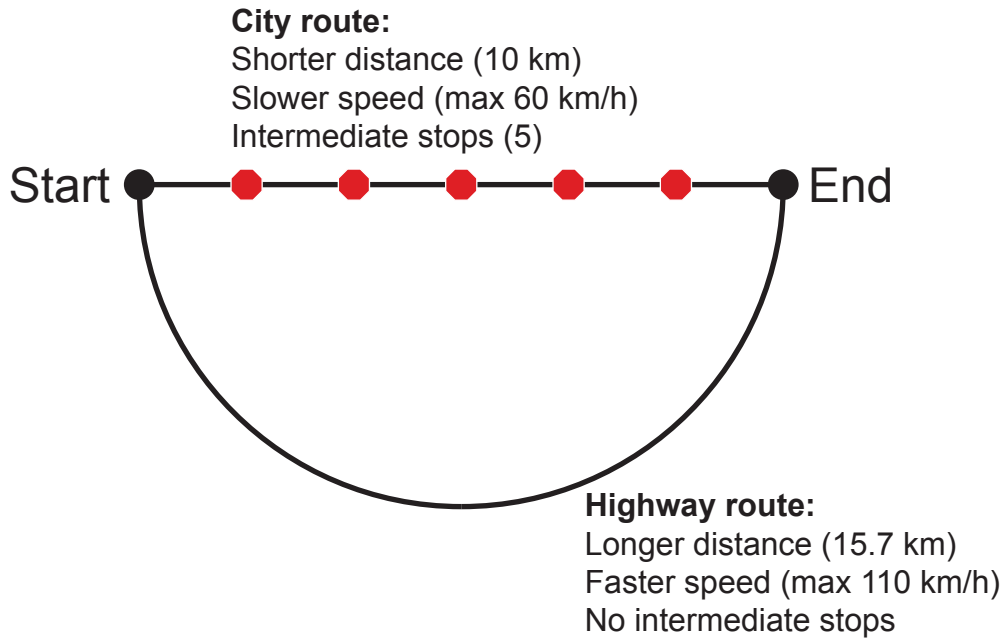


**Figure 7:** Observed and Predicted Relationship between fuel consumption and 2–15 m/s acceleration



Notes: The dark bars show the mean fuel consumption for different rates of acceleration from 2 to 15 m/s. The lighter bars show the mean predicted fuel consumption for driving at a constant speed of 15 m/s until a total distance (including the acceleration period) of 250 meters has been covered. The diamond markers show the predicted fuel consumption, for the acceleration period and for the entire 250 meters, for acceleration rates at the midpoint of each bar.

**Figure 8:** Summary of driving simulation model for policy comparisons



## A Price and mean driver fuel consumption

**Table 12:** Estimates for mean driver fuel consumption in liters per 100 kilometers

	(1)	(2)	(3)	(4)	(5)
n_price	-0.03 (0.071)	-0.07 (0.038)	-0.07 (0.032)	0.03 (0.021)	0.02 (0.022)
Acceleration events per km		0.93 (0.151)	0.72 (0.159)	1.29 (0.065)	1.16 (0.071)
Idle time (0–1)		0.47 (0.070)	0.35 (0.084)	0.60 (0.063)	0.44 (0.077)
Speed (km/h)		0.23 (0.125)	-0.03 (0.224)	0.17 (0.067)	-0.29 (0.145)
Speed > 100 km/h			-0.07 (0.114)		0.30 (0.089)
Speed   Speed > 100 km/h			0.20 (0.066)		0.01 (0.035)
Acceleration (m/s <sup>2</sup> )			0.18 (0.047)		0.14 (0.054)
Deceleration (m/s <sup>2</sup> )			0.04 (0.045)		0.01 (0.054)
Outside temperature (°C)	-0.33 (0.189)	-0.42 (0.076)	-0.46 (0.061)	-0.62 (0.072)	-0.61 (0.070)
Air conditioning (0–1)	0.05 (0.182)	0.26 (0.051)	0.28 (0.046)	0.43 (0.049)	0.40 (0.046)
Female (0/1)	0.73 (0.201)	-0.01 (0.080)	0.15 (0.077)		
Age 40–50 (0/1)	-0.68 (0.280)	-0.21 (0.102)	0.01 (0.110)		
Age 60–70 (0/1)	-0.65 (0.233)	-0.18 (0.100)	0.03 (0.127)		
Driver fixed effects	N	N	N	Y	Y
Observations	108	108	108	758	713
Adjusted R <sup>2</sup>	0.204	0.872	0.901	0.867	0.847

*Note:* See Table 2.

## B Physical model of fuel consumption

The theoretical structure of the physical model is developed from Saerens et al. (2010) and Hellström et al. (2009).<sup>24</sup> Unlike these papers in the engineering literature, we use our observed data on fuel consumption to econometrically estimate the parameters of the model for our particular vehicle type.<sup>25</sup>

Instantaneous fuel flow  $\dot{m}$  is modeled as a non-linear function of the engine rotation speed  $\omega$  and the engine torque  $T_e$ .<sup>26</sup> Both of these can be written as a function of vehicle speed and acceleration. First, conditional on gear, engine rotation speed is a linear function of the vehicle speed, as in Equation (3):

$$\omega = v \left( \frac{i_g}{R_w} \right) \quad (3)$$

In this equation  $R_w$  is the wheel radius and  $i_g$  is the combined transmission and final drive conversion ratio for the chosen gear  $g$ .

The vehicle driveline transmits the engine torque into a friction force on the wheels of the vehicle,  $F_w$ , as in Equation (4):

$$T_e = \frac{R_w}{i_g \eta} F_w \quad (4)$$

The power transmission efficiency,  $\eta$ , is assumed to be constant.

The equation of motion for the vehicle is then given by Equation (5):

$$F_w = ma + F_a(v) + F_r(\alpha) + F_N(\alpha) \quad (5)$$

In this equation  $m$  is the mass of the vehicle.  $F_a(v)$  is the aerodynamic resistance, which

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<sup>24</sup>Other papers that apply mathematical programming techniques to a simplified vehicle model in order to derive fuel-minimizing driving behavior include Schwarzkopf and Leipnik (1977), Chang and Morlok (2005) and Saerens et al. (2009).

<sup>25</sup>In a similar approach, Hooker (1988) uses a statistical model to simulate fuel consumption for 15 types of vehicle. Fuel consumption measurements are from dynamometer testing in a laboratory. These are matched to observations of engine speed and load during driving on a track. Ahn et al. (2002) also use a combination of laboratory measurements and field driving tests to develop models of vehicle fuel consumption and emissions. In contrast to this approach, we use simultaneous observations of speed, acceleration and fuel consumption from driving on real roads.

<sup>26</sup>This is the approach used by Saerens et al. (2010). An alternative is to model fuel flow as a function of the fueling level (determined by the driver using the accelerator pedal) and the engine speed (Hellström et al., 2009). This requires an additional equation to relate the engine torque, fueling level and engine speed.

is proportional to the square of the velocity of the vehicle.<sup>27</sup>  $F_r(\alpha)$  is the rolling resistance of the vehicle, which is proportional to the cosine of the roadway slope,  $\alpha$ .  $F_N(\alpha)$ , the gravitational force, is proportional to the sine of the road slope.

Combining Equations (4) and (5) allows us to write engine torque as a function (conditional on gear) of acceleration, the square of velocity, and the slope of the road. Although it is possible to estimate the entire model conditional on observed gear, for our counterfactual analysis we wish to abstract away from modelling gear changes by the automatic transmission. Instead we assume that gear changes are instantaneous and that  $i_g$  is itself a function of speed.

The final stylized expression for instantaneous fuel flow is given by Equation (6):

$$\dot{m} = f(\omega, T_e) = f(\omega(v, i_g(v)), T_e(v, a, \alpha, i_g(v))) \quad (6)$$

Saerens et al. (2010) use a polynomial approximation for this function, including the interaction of cubic terms in  $\omega$  and quadratic terms in  $T_e$ .

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<sup>27</sup>Other components of air resistance are the frontal surface of the vehicle, the vehicle drag coefficient, and density of air. These are assumed to be constant.